Course Introduction and Basics of Vectors and Matrices

Larry Caretto
Mechanical Engineering 501A

Seminar in Engineering Analysis

August 28, 2017

Northridge

Overview

- Discuss course outline, schedule, grading, office hours, etc.
- Discuss engineering problems
- Begin discussion of first topic, review of matrix and vector operations
  - Concept of vectors from mechanics
  - Use of matrices and vectors
  - Matrix representation of vectors
  - Determinants and matrix inverses

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Course Structure

- ME 501AB is a one-year course in engineering and numerical analysis
- Look at solving problems once they are formulated
- Two overall goals
  - Understand advanced mathematical and computational approaches encountered in your work and future course work
  - Develop the ability to apply appropriate problem-solving skills

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Course Materials

- Web site: http://www.csun.edu/~lcaretto/me501a
- Notes for tonight
  - Course outline
  - Notes on matrix analysis
  - Power point presentation
  - September 6 homework problems
- Course notes and homework assign-ments available for subsequent weeks
- Download materials prior to class

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Instructor and Course Data

- lcaretto@csun.edu (larry.caretto@csun.edu)
- Office: part-time faculty office JD 3338
- Office hours MW 6:00– 6:45 pm
- Text: Kreyszig, *Advanced Engineering Mathematics*, (10th edition) Wiley, 2011
- Grading based on homework (10%), two midterms (50%) and a final (40%)
- See grading criteria in outline

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Course Objectives

- Understand similarity of vectors and function expansions
- Read publications of applied engineering analysis that involve simultaneous linear equations, matrices, eigenvalues, ordinary differential equations, special functions such as Bessel functions, orthogonal functions, eigenfunction expansions.
- Be familiar with algorithms and software packages for matrix problems, and ordinary differential equations and understand the limitations of these approaches.

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Course objectives II

- Analyze engineering problems that require systems of simultaneous equations, understand why unique solutions may not be possible.
- Perform manipulations of matrices when this is appropriate for the analysis of engineering systems.
- Understand the role that eigenvalues and eigenvectors play in engineering analysis, obtain these quantities in simple cases, and use software to obtain them in systems that are more complex.

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Course objectives III

- Understand when solutions to ordinary differential equations are possible and obtain solutions in those cases.
- Obtain power series solutions to ordinary differential equations.
- Obtain solutions to ordinary differential equations that involve special functions such as Bessel functions.
- Be familiar with the use of Laplace transforms for solving ordinary differential equations and be able to use a transform table to get such solutions in simple cases.

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Course objectives IV (and last)

- Use various algorithms for solving systems of ordinary differential equations and understand the approaches used to keep the accuracy to the solution within the bounds desired by the user
- Solve differential equations applied to initial value problems, boundary value problems, and eigenvalue problems.
- Prepare for consideration of partial differential equations in ME 501B

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Course Operation

- Weekly homework assignments due Wednesdays at start of class
- Midterms on Wednesdays (10/18 & 11/15)
- Final on Monday, December 11 (8-10 pm)
- General lecture schedule modified at times by subject matter
  - Review previous material
  - Discuss new material
- Lectures notes posted online prior to class
  - Use to simplify notetaking

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Computing

- From time to time homework problems will require some computer use
- Students will generally be able to choose their computing platform
  - Excel, Excel with Visual Basic, MATLAB, C++, Fortran, etc.
  - Some assignments will be given in Excel and MATLAB
- ME department computers are available in EN 1118 and 1116
  - Excel/VBA, MATLAB and Visual C++

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Text and Homework

- Required text by Kreyszig is specified as 10th edition
  - Reading assignments in course outline are for both this edition and previous (9th) edition
  - Download homework problems from web site; independent of text edition
  - NO late homework assignments; homework solutions will be posted on line after due date

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Galileo Galilei

(1564-1642)

You cannot teach people anything; you can only help them find it within themselves.

http://space.about.com/od/astronomyhistory/a/galileoquotes.htm

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Goals for this Course

- My goal is to help all students find within themselves sufficient knowledge of engineering analysis so that they will all get an A grade in the course
- What is your goal for this course?
- What will you do to achieve that goal?

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How to get your A

- Spend six to ten hours per week outside class studying for the course
- Prepare for lecture and be ready to ask questions
  - Read the assigned reading before class
  - Download, print, and review the lecture presentations before class
    - Use these as notes so that you can follow the lecture; write additional notes on these presentations

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How to Get your A, Part II

- Study with fellow students and try to answer each other's questions
- Do the homework assignments
- Contact me by email or office visits to ask questions
- Develop a good working relation with other members the class
  - Help each other with homework
- Participate in class discussions

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What I will do to help

- Arrive at class a few minutes early to answer any questions you may have
- Give lectures that stress application of basics to problem solving
- Return homework and exams promptly so that you can learn from your errors
- Be available for questions during office hours and anytime by email
- Send entire class emails as appropriate

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Engineering Problems

- Look at problems written in terms of differential equations
- Complex mechanical systems and structures have simultaneous equations
- Engineering analysis of simple geometries shown overall behavior and trends
- Generalize vectors and functions
- Computer solutions use numerical analysis by finite difference or finite element methods
- Have to generate grid for the numerical solution that fits geometry

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Modeling Engineering Systems

- Based on simple physical laws
  - Conservation of mass
  - Force momentum balance (F = ma)
  - Energy conservation
  - Gradient relationships (Ohm, Hooke, Fourier)
- Use differential equations to account for variations over small spatial scales
- Models derived based on consideration of differential volume as size approaches zero

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Mechanical Systems

- Newton's second law, $\mathbf{f} = m\, d^2\mathbf{x}/dt^2$
- Have separate equation for each coordinate direction
- For linked systems and structures the displacements of various points are linked by mutual forces in shared members
- Leads to system of ordinary differential equations
- Find eigenvalues of matrix solutions give basic modes of vibration in system

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Diffusion/Heat Transfer

- Species concentrations, c, and temperature, T, follow the equation shown below
- Thermal conductivity, k, is material property that depends on material used
- $k\frac{d^2T}{dx^2} + \dot{Q}(x)=0$
- k is the thermal conductivity [W/(m•K)]
- $\dot{Q}(x)$ is the heat source (W/m^3)
  - For mass diffusion, T and k are replaced, respectively, by concentration, c, and diffusion coefficient, D; there is no mass source

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Stress in a Beam Under Load

- Straight beam, rectangular cross section
- Load applied along length of beam = w(x)
- EI is product of Young's modulus and moment of inertia
- Boundary conditions for beam fixed at both ends

$$EI\frac{d^4y}{dx^4} = w(x) \qquad y = \frac{dy}{dx} = 0 \; at \; x = 0 \; and \; x = L$$

- Other boundary conditions possible

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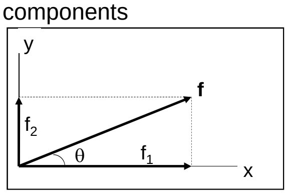
Vectors from Mechanics

- A vector has a magnitude and a direction
  - Can represent by components
- Two dimensional vector, **f**, with magnitude, |**f**| and direction θ



- **f** has components f_1 and f_2 in x and y directions; **f** = f_1**i** + f_2 **j**

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Vectors and Dot Products

- Represent **f** in 3D as **f** = f_1**i** + f_2**j** + f_3**k** or [f_1 f_2 f_3] components in x, y, z directions
- Unit vectors **i** = [1 0 0], **j** = [0 1 0], **k** = [0 0 1]
- Dot product of two vectors, **f•dx**, multiplies first vector by component of the second vector in direction of the first vector
- **f•dx** = **|f||dx|**cos(θ) (**|f|** is magnitude)
- Perpendicular vectors $\theta = \pi/2$ and **f•dx** = 0
- Parallel vectors have $\theta = 0$, **f•dx** = **|f||dx|**

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Vectors and Dot Products

- For perpendicular unit vectors **i**, **j**, and **k**, **i•i** = **j•j** = **k•k** = 1 and **i•j** = **j•k** = **i•k** = **j•i** = **k•j** = **k•i** = 0
- **f** = f_1**i** + f_2**j** + f_3**k** & **dx** = dx_1**i** + dx_2**j** + dx_3**k**
- Because **i**, **j**, and **k**, are orthonormal [orthogonal (mutually perpendicular) and normal (unit vectors)], we can write
- **f•dx** = $f_1dx_1 + f_2dx_2 + f_3dx_3$
- Is **f•dx** a vector? ***NO!***

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Matrix Basics

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & \cdots & a_{1m} \\ a_{21} & a_{22} & a_{23} & \cdots & \cdots & a_{2m} \\ a_{31} & a_{32} & a_{33} & \cdots & \cdots & a_{3m} \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & \vdots & & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & \cdots & a_{nm} \end{bmatrix}$$

- Array of numbers with n rows and m columns
- Components are $a_{(row)(column)}$
- Size of matrix (n x m) is number of rows and columns
  - A matrix with n = m is called a square matrix

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More Matrix Basics

- Two matrices are equal (*e.g.*, **A** = **B**)
  - If both **A** and **B** have the same size (rows and columns)
  - If each component of **A**,is the same as the corresponding component of **B** ($a_{ij} = b_{ij}$ for all i and j)
- A square matrix has the same number of rows and columns
- A diagonal matrix, **D**, has all zeros except for the principal diagonal

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Diagonal Matrix

$$\mathbf{A} = \begin{bmatrix} a_1 & 0 & 0 & \cdots & \cdots & 0 \\ 0 & a_2 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & a_3 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & a_n \end{bmatrix}$$

- The diagonal matrix **A** is a square matrix with nonzero components only on the principal diagonal
- Components of **A** are $a_i\delta_{ij}$, where δ_{ij} is the Kroenecker delta

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

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Matrix Operations

- Can add or subtract matrices **only** if they are the same size
  - **C** = **A** ± **B** only valid if **A**, **B**, and **C** have the same size (rows and columns)
  - Components of **C**, $c_{ij} = a_{ij} \pm b_{ij}$
- Multiplication by a scalar: **C** = x**A**
  - **C** and **A** have the same size (rows and columns)
  - Components of **C**, $c_{ij} = xa_{ij}$

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Matrix Operations Quiz

$$\mathbf{A} = \begin{bmatrix} 3 & 12 & -6 \\ 14 & -2 & 0 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -6 & 2 & 11 \\ 18 & 0 & 5 \end{bmatrix}$$

- Find **A + B**

$$\mathbf{A}+\mathbf{B} = \begin{bmatrix} 3 & 12 & -6 \\ 14 & -2 & 0 \end{bmatrix} + \begin{bmatrix} -3 & 2 & 11 \\ 18 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 14 & 5 \\ 32 & -2 & 5 \end{bmatrix}$$

- Find (3/2) **A**

$$\frac{3}{2}\mathbf{A} = \frac{3}{2}\begin{bmatrix} 3 & 12 & -6 \\ 14 & -2 & 0 \end{bmatrix} = \begin{bmatrix} \frac{9}{2} & 18 & -9 \\ 21 & -3 & 0 \end{bmatrix}$$

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Null (0) and Unit (I) Matrices

- For any matrix, **A**, **A** + **0** = **0** + **A** = **A; IA** = **AI** = **A** and **0 A** = **A 0** = **0**
- The unit (or identity) matrix is a square matrix; the null matrix need not be square

$$\mathbf{0} = \begin{bmatrix} 0 & 0 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & 0 & & & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & 0 \end{bmatrix} \qquad \mathbf{I} = \begin{bmatrix} 1 & 0 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & 1 \end{bmatrix}$$

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Transpose of a Matrix

- Transpose of **A** denoted as $\mathbf{A}^T$
- Reverse rows and columns; for **B** = $\mathbf{A}^T$
  - $b_{ij} = a_{ji}$
  - If **A** is (n x m), **B** = $\mathbf{A}^T$ is (m by n)

$$\mathbf{A} = \begin{bmatrix} 3 & 12 & -6 \\ 14 & -2 & 0 \end{bmatrix} \qquad \mathbf{A}^T = \begin{bmatrix} 3 & 14 \\ 12 & -2 \\ -6 & 0 \end{bmatrix}$$

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Row and Column Vectors

- Matrices with only one row or only one column are called row or column vectors (or matrices)

$$\mathbf{r} = [r_{11} \quad r_{12} \quad r_{13} \quad \cdots \quad \cdots \quad r_{1m}] = [r_1 \quad r_2 \quad r_3 \quad \cdots \quad \cdots \quad r_m]$$

$$\mathbf{c} = \begin{bmatrix} c_{11} \\ c_{21} \\ c_{31} \\ \vdots \\ \vdots \\ c_{n1} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ \vdots \\ c_n \end{bmatrix}$$

- Single subscript is usual notation, but implied "1" for single row or column is important for two-subscript formulas

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Matrix Multiplication Preview

- Not an intuitive operation. Look at two coordinate transformations as example

$$y_1 = a_{11}x_1 + a_{12}x_2 \qquad z_1 = b_{11}y_1 + b_{12}y_2$$
$$y_2 = a_{21}x_1 + a_{22}x_2 \qquad z_2 = b_{21}y_1 + b_{22}y_2$$

- Substitute equations for y in terms of x into equations for z_1 and z_2

$$z_1 = b_{11}[a_{11}x_1 + a_{12}x_2] + b_{12}[a_{21}x_1 + a_{22}x_2]$$
$$z_2 = b_{21}[a_{11}x_1 + a_{12}x_2] + b_{22}[a_{21}x_1 + a_{22}x_2]$$

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Matrix Multiplication Preview II

- Rearrange last set of equations to get direct transformation from x to z

$$z_1 = [b_{11}a_{11} + b_{12}a_{21}]x_1 + [b_{11}a_{12} + b_{12}a_{22}]x_2 = c_{11}x_1 + c_{12}x_2$$
$$z_2 = [b_{21}a_{11} + b_{22}a_{21}]x_1 + [b_{21}a_{12} + b_{22}a_{22}]x_2 = c_{21}x_1 + c_{22}x_2$$

$$c_{11} = [b_{11}a_{11} + b_{12}a_{21}] \qquad c_{12} = [b_{11}a_{12} + b_{12}a_{22}]$$
$$c_{21} = [b_{21}a_{11} + b_{22}a_{21}] \qquad c_{22} = [b_{21}a_{12} + b_{22}a_{22}]$$

$$c_{ij} = \sum_{k=1}^{2} b_{ik}a_{kj} \qquad (i = 1,2;\ j = 1,2)$$

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Matrix Multiplication Preview III

- Coefficients as matrix components

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} b_{11}a_{11} + b_{12}a_{21} & b_{11}a_{12} + b_{12}a_{22} \\ b_{21}a_{11} + b_{22}a_{21} & b_{21}a_{12} + b_{22}a_{22} \end{bmatrix}$$

$$c_{ij} = \sum_{k=1}^{2} b_{ik}a_{kj} \qquad (i = 1,2;\ j = 1,2)$$

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General Matrix Multiplication

- For matrix multiplication, **C** = **AB**
  - **A** has n rows and p columns
  - **B** has p rows and m columns
  - **C** has n rows and m columns
  - ***Left matrix columns = right matrix rows***
  - ***Product matrix rows = left matrix rows***
  - ***Product matrix columns = right matrix cols***

$$c_{ij} = \sum_{k=1}^{p} a_{ik}b_{kj} \qquad (i = 1, n;\ j = 1, m)$$

- Example $\mathbf{A} = \begin{bmatrix} 3 & 0 & -6 \\ 4 & -2 & 0 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 3 & 4 \\ 1 & 2 \\ 6 & 1 \end{bmatrix}$

$$\mathbf{AB} = \begin{bmatrix} 3(3)+0(1)-6(6) & 3(4)+0(2)-6(1) \\ 4(3)-2(1)+0(6) & 4(4)-2(2)+0(1) \end{bmatrix} = \begin{bmatrix} -27 & 6 \\ 10 & 12 \end{bmatrix}$$

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Matrix Multiplication Quiz

- For matrix multiplication, **C** = **BA**
  - **B** has n rows and p columns
  - **A** has p rows and m columns
  - **C** has n rows and m columns

$$c_{ij} = \sum_{k=1}^{p} b_{ik}a_{kj} \qquad (i = 1, n;\ j = 1, m)$$

- Reexamine previous example

$$\mathbf{B} = \begin{bmatrix} 3 & 4 \\ 1 & 2 \\ 6 & 1 \end{bmatrix} \qquad \mathbf{A} = \begin{bmatrix} 3 & 0 & -6 \\ 4 & -2 & 0 \end{bmatrix}$$

Class Exercise: Find BA

- Can we find **BA**?
- Yes – the number of columns in **B** is the same as the number of rows in **A**

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Exercise Solution

- For matrix multiplication, **C** = **BA**
  - **B** has 3 rows and 2 columns
  - **A** has 2 rows and 3 columns
  - **C** has 3 rows and 3 columns

$$c_{ij} = \sum_{k=1}^{2} b_{ik}a_{kj} \qquad (i = 1,3;\ j = 1,3)$$

$$\mathbf{B} = \begin{bmatrix} 3 & 4 \\ 1 & 2 \\ 6 & 1 \end{bmatrix} \qquad \mathbf{A} = \begin{bmatrix} 3 & 0 & -6 \\ 4 & -2 & 0 \end{bmatrix}$$

$$\mathbf{BA} = \begin{bmatrix} (3)(3)+(4)(4) & (3)(0)+(4)(-2) & (3)(-6)+(4)(0) \\ (1)(3)+(2)(4) & (1)(0)+(2)(-2) & (1)(-6)+(2)(0) \\ (6)(3)+(1)(4) & (6)(0)+(1)(-2) & (6)(-6)+(1)(0) \end{bmatrix} = \begin{bmatrix} 25 & -8 & -18 \\ 11 & -4 & -6 \\ 22 & -2 & -36 \end{bmatrix}$$

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